Brachistochrone problem pdf

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The rapid descent curve is not a straight or polygonal line (blue), but a cycloid (red). In mathematics and physics, the brachistochron curve (from the ancient Greek βράχιστος χρόνος (brachistos chronos) shortest time), or the curve of the rapid descent, is the curve lying on the plane between point A and bottom B, where B is not directly below A, on which the ball slides without friction under the influence of a uniform gravitational field to a certain endpoint in no time. The problem was posed by Johann Bernoulli in 1696. The brachistochron curve is the same shape as the taustochron curve; both are cycloids. However, some of the cycloids used for each of the two varies. More precisely, the brachistochron can be used until the full rotation of the cycloid (at the limit when A and B are on the same level), but always starts on the threshold. In contrast, the taupochron problem can only be used until the first half of the rotation, and always ends in horizontal. The problem can be solved with tools from calculus variations and optimal control. The curve does not depend on both the weight of the test body and the local gravity. Only the setting is chosen, so that the curve corresponds to the starting point A and the point of completion of B. Johann Bernoulli's story posed a brachistory problem to the readers of Acta Eruditorum in June 1696. He said: I, Johann Bernoulli, speak to the most brilliant mathematicians in the world. There is nothing more attractive to smart people than an honest, complex problem, the possible solution of which will give fame and remain a lasting monument. Following the example of Pascal, Fermat, etc., I hope to receive the gratitude of the entire scientific community by presenting the best mathematicians of our time with a problem that will test their methods and the strength of their intellect. If someone tells me a solution to the proposed problem, I will publicly declare it worthy of praise Bernoulli wrote a statement about the problem as: Given the two points A and B in the vertical plane, that curve is traced to the point operated only by gravity, which begins in A and reaches B in no time. Johann and his brother Jacob Bernoulli received the same decision, but Johann's conclusion was wrong, and he tried to pass Jacob's decision as his own. Johann published the decision in the journal in May next year and noted that the decision has the same curve as the taugochron Curve Of Juygens. After receiving a differential equation for the curve method given below, he continued to show that it was giving a cycloid. However, his evidence is overshadowed by his use of one constant instead of three constants, vm, 2g and D, below. Bernoulli allowed six months to deal with, but none was during this period. At Leibniz's request, the time was publicly extended for a year and a half. On January 29, 1697, at 4 p.m., when he returned home from the Royal Mint, Isaac Newton found the call in a letter from Johann Bernoulli. Newton stayed up all night to resolve the issue, and sent the decision anonymously on the following message. After reading the decision, Bernoulli immediately recognized its author, exclaiming that he recognizes the lion from his claws. This story gives some insight into Newton's power, as it took Johann Bernoulli two weeks to resolve it. Newton also wrote, I don't like being loved, being huddled and teased by foreigners about mathematical things... and Newton has already solved Newton's minimum resistance problem, which is considered the first of its kind in calculating variations. In the end, five mathematicians responded with decisions: Newton, Jacob Bernoulli, Gottfried Leibniz, Ehrenfried Walter von Tsyrnhaus and Guillaume de l'Hepital. Four of these solutions (with the exception of l'H'pital's) were published in the same edition of the magazine as Johann Bernoulli. In his work, Jacob Bernoulli gave evidence of a condition at least similar to what is below before showing that his solution is a cycloid. According to Newtonian scientist Tom Whiteside, in an attempt to outdo his brother, Jacob Bernoulli created a tougher version of the problem of the brachistory. In addressing it, he developed new methods that were perfected by Leonhard Euler in what the latter called (in 1766) calculus variations. Joseph-Louis Lagrange did further work that led to today's infinitely low calculus. Earlier, in 1638, Galileo tried to solve a similar problem for the path of rapid descent from point to wall in his two new sciences. He concludes that the arc of the circle is faster than any number of its chords, from the previous one we can conclude that the fastest way of all lationem omnium velocissimam from one point to another is not the shortest path, but a straight line, but an arc of a circle. ... Therefore, the closer the landfill is inscribed to the circle, the shorter the time it takes to descend from A to C. What has been proven for the quadrant is true for small arcs; reasoning is the same. Immediately after Theorem 6 of two new sciences, Galileo warns of possible misconceptions and the need for higher science. In this dialogue, Galileo examines his work. The real solution to Galileo's problem is half of the cycloid. Galileo studied the cycloid and gave it its name, but the connection between him and his problem had to wait for achievements in mathematics. The decision of Johann Bernoulli Direct Method In a letter by Henri Basnage, held at the University of Basel Public Library, dated March 30, 1697, Johann Bernoulli stated that he had found two methods (always called direct and indirect) to show that brachistochron was a common cycloid, also called Following Leibniz's advice, he included only an indirect method in Acta Eruditorum Lipsidae in May 1697. He wrote that this was partly because, in his view, it was sufficient to convince anyone who doubted the conclusion, in part because he had also solved two known optics problems that the late Mr. Guygens had raised in his treatise on light. In the same letter, he criticized Newton for concealing his method. In addition to his indirect method, he also published five other responses to the problem he had received. Johann Bernoulli's direct method is historically important, as it was the first proof that brachistochron is a cycloid. The method is to determine the curvature of the curve at each point. All other evidence, including Newton (which was not identified at the time) are based on finding a gradient at each point. It was not until 1718 that Bernoulli explained how he solved the problem of brachistochron by his direct method. He explained that he had not published it in 1697 for reasons that were no longer used in 1718. This article was largely ignored until 1904, when the depth of the method was first assessed by Constantine Karateodori, who stated that it shows that the cycloid is the only possible curve of rapid descent. According to him, other decisions simply implied that the descent time is stationary for the cycloid, but not necessarily minimally possible. The analytical solution of the Body is seen as sliding over any small Ce circular arc between the KC and Ke radii, with the K fixed center. The first stage of the evidence involves finding a specific circular arc, mm, which the body crosses at a minimum time. The KNC line crosses the AL to N, and the Kne line crosses it to n, and they make a small CKe angle on K. Let NK A and identify the variable point, C on KN extended. Of all the possible circular arcs of Ce, it is necessary to find the Arc mm, which requires a minimum time to slide between 2 radii, KM and km. To find Mm Bernoulli claims the following. Let MN and x. It defines m so that MD and MX, and so that mm and nx nx na and notes that x is the only variable, and that m is finite and infinitely small. A short time to travel in the MM arc is M m M D 1 x 2 (x x) (1 2 display {1}{2} frak mm MHMF frak {1}{2} . . . . . that because the mm is so small, the speed along it can be considered speed on M, which is the square root of the MD, the vertical distance of M below the horizontal line of AL. It follows that when used is differentiated it should give (x x x x 2 x 2 x 0 display frac (x-a)dx 2x {3}{2} This condition defines the curve by which the body glides in no time. For each point, M on a curve, curvature radius, MK is cut into 2 equal parts of its AL axis. unique to cycloids. Finally, it considers a more general case where the speed is an arbitrary function of X (x), so the time to be minimized is the (x) X display frac (x)). The minimum condition is X q (x) d d x (displaystyle X'frac) (x'a)dX'dx, which he writes as : X q q x displaystyle X (x'a) and which gives MN (x) as a function of NK (a). From this equation the curve could be obtained from the whole calculus, although it does not demonstrate it. Synthetic Solution He then continues what he called his synthetic solution, which was classic, geometric proof that there is only one curve that the body can slide down at a minimum time, and that the curve is cycloid. Suppose AMMB is part of the cycloid joining A to B, which the body slides down at a minimum time. Let the ICcJ be part of another B accession curve that may be closer to AL than AmmB. If the mm arc tilts the angle of the MKm in the center of the curvature, K, let the arc on the IJ that subtends the same angle cc. The circular arc through C with the center of K is Ce. Point D on AL vertically above M. Join K to D and H, where CG crosses KD, extended if necessary. Let the displaystyle and t be the time it takes for the body to fall along with Mm and Xie respectively. ∝ M M D 1 2 display tau propto frac mmmfrac {1}{2} , t ∝ C e C G 1 display tpropto frac ceCGfrac {1}{2} , Expand CG, to indicate F, where, C F - C H 2 M D (display CF' frac (CH-{2}) and since M m C e - M D C H (frac)Frac (Mm'Ce) frac (MD)CH, it follows that t . (C G M D) 1 2 (C G C F) 1 2 display (CG-MD) frak {1}{2} on the left ((CG) right) frak {1}{2} C MN and NK, for cycloids: G H and M D . H D D K and M D . C M M K (GH-frac display) MD. HDDK-frak (MD. CMMMSM, SH and M. C K M K and M D . (M K and C M) M K display STYLE CHSFRAK MD. KPC ICPCHRA (MD.) IFFCM, and C G and C H (M K and 2 C M) M K display style CGCHGHfrac (MD.) MKS2SM) If Ce is closer to K than mm, then C H and M D . (M K and C M) M K (CH'frac) (MD.) MK-CM((M K 2 C M) M K display style CGCH-GHfrac (MD.) MK-2SM) In any case, C F and C H 2 M D (CF display) frac CH {2} MD t' displaystyle q tau'gt; if if the arc, cc' subtended by the angle' infinitesimal' angle' mkm' on q ij' is not circular, it should be since the cec becomes a' right-triangle' in the' limit' as ' angle' mkm' approaches q zero.' note, bernoulli' proves that's a cf'gt;on a similar, but also another argument. From this he concludes that the body crosses the cycloid AMB in less time than any other ACB curve. Indirect method In accordance with the Principle of Fermat, the actual path between the two pointsthe ray of light is the one that takes the least time. In 1697, Johann Bernoulli used this principle to produce a brachistochron curve, examining the trajectory of the light beam in an environment where the speed of light increases after constant vertical acceleration (that of the gravity of d). To conserve energy, the instantaneous velocity of the body v after the fall of height in the uniform gravitational field is given: v q 2 g y displaystyle vsqrt , the speed of movement of the body on an arbitrary curve does not depend on horizontal movement. Bernoulli noted that the law of refraction gives a constant of movement for a beam of light in a variable density environment: sin θ v No 1 v d d d s 1 v m . {1}'v'frac (dx'ds' frac {1} v_'m', where vm is constant and θ the display theta represents a trajectory angle relative to θ the vertical. that the particle (or beam) with coordinates (x,y) moves away from the point (0.0) and reaches top speed after falling vertical distance D: v m 2 g D displaystyle v_'m'sqrt . The permutation of terms in the law of refraction and quadring gives: v m 2 d x 2 v 2 d s 2 v 2 (d x 2 q d y 2 {2} {2} {2} v_)ds'{2}'v'v'{2} (dx-{2}'dy'{2}), which can be solved for dx in terms of dy: d x q y v 2 v 2 (dx-{2}'{2}) that can be resolved for dx in terms of dy: d x q y v y 2 v 2 (d'{2}'{2}) (dx-{2} ,dy'sqrt (v_-{2}-{2}) . Replacement from expressions for v and vm above gives: d x q y D qy d y , displaystyle dxsqrt (frac) y'D-y,dy, which is a differential equation of an inverted cycloid generated by the D'2r diameter circle, the parametric equation of which: x y p (φ - sin φ φ) Displaystyle beginning is aligned x'r (Warfi - sin warfi)y'r (1-cos varphi end aligned), where the φ is a real parameter, corresponding to the angle through which the sliding circle rotates. For this φ the center of the circle is on (x, y) (rφ, r). In the problem of brachistochron, the movement of the body is given by the time of evolution of the parameter: φ (t) t, q t, g'displaystyle (t)'omega t',,,'omega s sqrt (frac s g'r) where t time since the release of the body (0.0). The decision of Jacob Bernoulli, brother of Johann Jakob, showed how the 2nd differentials can be used to obtain the condition in the least time. The upgraded version of the evidence is as follows. If we make a slight deviation from the path of the least time, then for the differential triangle formed by displacement along the path and horizontal and vertical movements, d s 2 and d x 2 d y Display style ds{2}'dx{2} dy {2}. On differentiation with dy fixed we get, 2 d d 2 s and 2 d d 2 x displaystyle 2ds d'{2}s'2dx' d'{2}x. . И, наконец, перестановка терминов дает, d x d d d 2 x d 2 s - v d 2 t displaystylefrac (dx'ds'ds'd'd'd{2}x'd'{2}s'v'd'{2}t), где последняя часть является перемещением для данного изменения во времени для 2-го дифференциала. Now consider changes along the two adjacent paths in the image below, for which horizontal separation between paths along the central d2x line (the same for the upper and lower differential triangles). Along old and new paths, parts, which are different are, d 2 t 1 th 1 v 1 d x 1 d 1 d 2 x display d{2}t_{1} frac {1} v_{1} frac (dx_{1}) ds_{1} d'{2}x 2 t 2 2 2 1 v 2 d x 2 d 2 x display d{2}t_{2}frac {1} v_{2} frac dx_{2} ds_{2} d'{2}x For way at least once these times are equal, so for their difference we get, d 2 t 2 x 2 t 1 x 0 (1 v 2 d x 2 d s s 2 1 1th 1 d x 1 d 1 d 1) d 2 x displaystyle d'{2}t_{2} d'{2}t_{1}'0'bigg Frak {1} v_{2} frak (dx_{2} ds_{2})-frak {1} v_{1}-dx_{1} ds_{1} {2} , 1 v 2 d x 2 d s 2 1 1 v 1 d x 1 d s 1 display frac {1} v_{2} frak (dx_{2} ds_{2} frak {1} v_{1} frak dx_{1} ds_{1}, which agrees with Johann's assumption, Newton's decision Introduction In June 1696 Johann Bernoulli used the pages of Acta Eruditorum Lipsidae to create a problem for the international mathematical community: find the shape of the curve by connecting the two fixed dots, so that the mass will slide down it, under the influence of gravity alone, in a minimal amount of time. The decision was originally expected to be submitted within six months. At Leibniz's suggestion, Bernoulli extended the call until Easter 1697, using a printed text titled The Program published in Groningen, Netherlands. The program is dated January 1, 1697 in the Gregorian calendar. It was December 22, 1696 in the Julian calendar, in use in the UK. According to Newton's niece, Katherine Conduit, Newton learned of the problem at 4 p.m. on January 29 and decided it by 4am the next morning. His decision, brought to the attention of the Royal Society, is dated January 30. This decision, later published anonymously in Philosophical Transactions, is correct, but does not indicate the method by which Newton came to his conclusion. Bernoulli, writing to Henri Basnaga in March 1697, pointed out that although his author, excessively modesty, had not disclosed his name, but even from the scant details provided he could be recognized as Newton's work as a lion of his claw (in Latin, tanquam ex ungue leonem). John Wallis, who was 80 at the time, learned about the problem in September 1696 from His younger brother Johann Bernoulli Hieronymus and spent three months trying to find a solution before it in December to David Gregory, who also failed to resolve it. After Newton presented his decision, Gregory asked for his details and took notes from their conversation. They can be found in the University of Edinburgh Library, manuscript A 78 1 display 78 {1}, dated 7 March 1697. Either Gregory didn't understand Newton's argument, or Newton's explanation was very brief. However, it is possible, with a high degree of certainty, to build Newton's proof of Gregory's notes, similar to his method, to determine solid minimum resistance (Principalia, Book 2, Proposition 34, Scholium 2). A detailed description of his solution to this last problem is included in the draft letter in 1694, also to David Gregory. In addition to the problem of the minimum time curve was the second problem, which Newton also solved at the same time. Both decisions appeared anonymously in the philosophical transactions of the Royal Society, in January 1697. The Brachistochrone Fig. 1 problem shows Gregory's chart (except for an additional IF line missing from it, and at the start point was added). The SVA curve is a cycloid and CHV is its generating circle. Since it seems that the body is moving upwards from e to E, it should be assumed that the small body is released from and slides along the curve to A, without friction, under the influence of gravity. Consider the small eE arcs that the body rises. Suppose it crosses a straight line eL to indicate L, horizontally shifted from E for a short distance, o, instead of an eE arc. Note that eL is not tangent on e, and that o will be negative when L is between B and E. Draw a line through E parallel TO CH, cutting eL on n. From the cycloid property, En is normal on tangent on E, and similarly tangent on E parallel VH. Since the offset, the EL is small it is little different in direction from the tangent on E so that the EnL angle is close to the right corner. At the limit, as the eE arc approaches zero, eL becomes a parallel VH, provided that o is small compared to eE, making the EnL and CHV triangles similar. In addition, en is approaching the length of the eE chord, and increasing length, e L and e and n L C H C V (eL-eE'nL' frac (o.CH)CV display, ignoring terms in o 2 (o'{2} display) and above, which represent an error due to the approximation that eL and VH parallel speed along eE or eL can be taken as on E, proportional to the C B S like CH, as C H and C B. C V (CH'sqrt display) (CB). That seems to be all that contains Gregory's note. Let t be extra time to reach L, t ∝ n L C B and o. C H C V . C B -o C v display tpropto frac nLsqrt (CB)frac o.CHCV. sqrt (CB) frac osqrt (CV) So the increase in time for passing a small arc moved to one endpoint depends only on the end point. Arc. However, by Newton's method, this is simply a condition required for the curve to pass at the lowest possible time. Therefore, he concludes that the minimum curve should be a cycloid. He says the following. Assuming it's now that rice. 1 is the minimum curve yet to be defined, with the vertical CV axis, and the CIRCLE CHV removed, and the rice. The two shows a part of the curve between the infinitely small eE arc and the further infinitely small Ff arc the final distance along the curve. The extra time, t to pass eL (rather than eE) is divided by speed on E (proportionate to C B stvrt CB) ignoring the terms in o 2 displaystyle o {2} and above: t ∝ o . D E e E . C B display tpropto frac o.DEeE. Sqrt CB , In L particle continues along the path of LM, parallel to the original EF, to some arbitrary point M. Since it has the same speed on L as in E, the time to pass the LM just as it would along the original EF curve. On M it returns to the original path at point f. By the same reasoning, shortening the time, T to reach F from M, not from F is T ∝ o . F G F f . C I display T'propto (t - T) is an additional time required along the eLMf path compared to the original eEFf : (t t) ∝ (D E e E C B - F F F C C I). o display (t-T) (display o on the left (frak DEeE'sqrt CB-frac FG Ffsqrt CI (right) plus terms in o 2 display style o {2} and above (1) Because eEFf is a minimum curve, t - T) should be more than zero, whether positive or negative. It follows that the ratio o in (1) should be zero: D E e E C B - F G F F C I (frac)DE'eE' sqrt (CB) -frac (FG)Ff'sqrt (CI) (2) in the limit of both eE and fF. Note that eEFf is a minimum curve, it should be assumed that the ratio o 2 (displaystyle o'{2}) is more than zero. Obviously, there must be 2 equal and opposite movements, or the body will not return to the end point, A, curve. If e is fixed, and if f is considered a variable point above the curve, then for all such points, f, F F F f C I display frac FGFfsqrt (CI) is permanent (equal D E E E E C B display frac DE Keeping F fixed and make e variable clear that D E E E E C B display because points, e and f are arbitrary, the equation (2) can only be true if D E e E C B is a permanent display (frac)DE'eE'sqrt (CB) text-constant, everywhere, and this condition characterizes the curve that is sought. This is the same method that he uses to find the form of Solid Least Resistance. For Cycloid, D E e E and B H V H C B (display) frak (DESE) sqrt (CB) frac (CH'CV.) sqrt (CB), which was shown be permanent, and the brachistochron is a cycloid. Newton gives no indication of how he discovered that cycloid satisfied this latter attitude. This may have been through trial and error, or he may have immediately acknowledged that this implied the curve was a cycloid. See also the Mathematics portal Physics portal Aristotle Wheel Paradox Beltrami Identity Calculus Variations Catenary Cycloid Newton's Minimal Resistance Problem Tautochrone Curve Trochoid Evenly Accelerated Motion Links - Chisholm, Hugh, Ed. Brachistochron. Encyclopedia Britannica (11th place). Cambridge University Press. Stuart, James. Section 10.1 - Curves defined by parametric equations. Calculus: Early Transcendents. 7th o.p. Belmont, Cay.: Thomson Brooks/Cole, 2012. Print. Ross, I.M. Paradigm of the Brachistochron, in the primer on the principle of Pontryagin in optimal control, collegiate publishers, 2009. ISBN 978-0-9843571-0-9. Hand B, Louis N. and Janet D. Finch. Chapter 2: Variation Calculus and its application to mechanics. Analytical mechanics. Cambridge: Cambridge UP, 1998. 45, 70. Print. Johann Bernoulli (June 1696) Problema novum ad cujus solutionem Mathematici invitantur. (A new problem to which mathematicians are invited.), Acta Eruditorum, 18 : 269. From page 269: Dastis in plano verticali duobus punctis A and B (vid Fig. 5) appoint Mobili M, viam AMB, for quam gravitate sua descendens - moveri incipiens a puncto A, brevissimo tempore perveniat ad alterum punctum B. (Given in the vertical plane two points A and B (see figure 5), appoint the body of the AMB's path, by which - going down with its own weight and starting to move by gravity from point A - it will arrive at another point B in no time.) Solutions to the problem of Johann Bernoulli 1696: Isaac Newton (January 1697) De ratione temporis quo grave labitur per rectam data duo puncta conjungentem, ad tempus brevissimum quo, vi gravitatis, transit ab horum uno ad alterum per arcum oidcyclis (On the evidence that the time in which weight slips along the line Joining the two points is the shortest in terms of time when it passes through gravitational force, from one of these dots to the other through the cycloid arc), the Philosophical Deals of the Royal Society of London, 19 : 424-425. G.G.L. (Gottfried Wilhelm Leibniz) (May 1697) Communicatio suae pariter, duarumque alienarum ad edendum sibi primum a Dn. Jo. Bernoullio, deinde a Dn. Marchione Hospitalio communicatarum solutionum problematis curva celerrimi descensus a Dn. Jo. Bernoullio Geometris publice propositi, una cume sua problematis alterius ab eodem postea. (His communication along with the other two in the report to him first from Johann Bernoulli, and then from the Marquis de l'H'H'H'pital, reported solving the problem of the rapid descent curve, (which was) publicly proposed by Johann Bernoulli, a geometer - one with the solution other problems subsequently proposed by the same man), Acta Eruditorum, 19 : 201-205. Johann Bernoulli (May 1697) Curvatura radii in diaphanis non uniformibus, Solutioque Problematis se in Actis 1696, p. 269, propositi, de invenienda Linea Brachystochrona, id est, in qua grave dato puncto ad datum punctum breviss tempoimore decurrit, decurva Synchrona seu radiorum unda construenda. (The curvature of the rays in non-homogeneous media, and the solution to the problem I proposed in Acta Eruditorum 1696, p. 269, from which you can find the line of brachistochron (i.e. curve), that is, in which the weight descends from this point at the moment in the shortest possible time, and in the construction of the tautochron or wave of rays of light.), Act Eruditorum, 19 : 206-211. Jacob Bernoulli (May 1697) Solutio problematum fraternorum, ... (Solving my brother's problems, ...), Acta Eruditorum, 19 : 211-214. Marquis de l'Hepital (may 1697) Dominie Marchionnis Hospitalii solutio problematis de linea celeri descensus (Solution by Lord Marquis de l'Hepital problems of the rapid origin line), Acta Eruditorum, 19 : 217-220. reissued by Isaac Newton (May 1697) Excerpta ex Transactionibus Philos. Anglic. M. Jan. 1697. (Excerpt from English Philosophical Deals in January 1697), Acta Eruditorum, 19 : 223-224. 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Table IV from Bernoulli's article in Acta Eruditorum 1697 Brachistochrones by Michael Trott and Brachistochrone Problem Okey Arik, Wolfram Demonstrations Project. The problem of the brachistor in MacTutor Geodesics Revisited - Introduction to geodesy, including two ways to obtain a geodesy equation with brachistochron as a special case of geodesy. The best solution to the problem of brachistochron in Python. Direct line, nursery, brachistochron, circle, and Fermat Unified approach to some geodesy. Extracted from the

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